MTH401 27 Feburary
2012 Final term PAPER SOLVED
TODAY's Paper

Total Question: 52
Mcqz: 40
Subjective question: 12
4 q of 5 marks
4 q of 3 marks
4 q of 2 marks

Guidelines:
Prepare this file as I included all past papers and current papers (shared till now) in it. You will have to clear the concepts and formulas of topics according to which questons are solved in file.

TODAY's PAPER no 1

Objective: MCQz

Topic	Number of Mcqz
Ratio Test Convergence Divergence	5
D.E(Integrating Factors +Homogenous+linear+bernoli)	**7**
Z= $\sqrt{X^2 + Z^2}$	**1**
Reactance & Impedence	**1**
Damped Motion	**2**
Maxima	**1**
Quasi period	**3**
Besslen's Equation	**1**
Matrix Type(square+system to matrix conversion)	**6**
Eigen Values+Eigen Vector	**4**
Multiplicy of Eigen Vector	**3**

D.E operator	**2**
General Solution	**1**
BVP	**1**

Please review the formulas of above topics.

Q:1

$$2\frac{dx}{dt} - 5x + \frac{dy}{dt} = 5e^{t}$$
$$\frac{dx}{dt} - x + \frac{dy}{dt} = e^{t}$$

lec 36 example 1

in decoupled form.

$$2\frac{dx}{dt} - 5x + \frac{dy}{dt} = 5e^t$$
$$\frac{dx}{dt} - x + \frac{dy}{dt} = e^t$$

$$(2D-5)x + Dy = 5e^t$$
$$(D-1)x + Dy = e^t$$

Determinants are $\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix}$, $\begin{vmatrix} 5e^t & D \\ e^t & D \end{vmatrix}$, $\begin{vmatrix} 2D-5 & 5e^t \\ D-1 & e^t \end{vmatrix}$

Therefore, in decpoupled form, we get

$$\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix} x \quad \begin{vmatrix} 5e^t & D \\ e^t & D \end{vmatrix}$$

$$\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix} y \quad \begin{vmatrix} 2D-5 & 5e^t \\ D-1 & e^t \end{vmatrix}$$

Q:2

Find order of homogenous equation obtained from non homogenous differential equation:

$y''+4y'+3y=4x^2+5?$ **? (2 MARKS)**

Find the eigenvalues of the following system

$$X' \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

Solution:

$$X' \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

$$A \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A-\lambda I|=0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$$

$(3-\lambda)(-3-\lambda)+36=0$

$3(-3-\lambda)-\lambda(-3-\lambda)+36=0$

$-9-3\lambda+3\lambda+\lambda^2+36=0$

$-9+\lambda^2+36=0$

$\lambda^2+27=0$

$\lambda=\sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

Q:3

What is Chemical reaction first order equation? (2) Page no 100

Answer:

$$\boxed{\frac{dX}{dt}=k\,X}$$

$k<0$ because X is decreasing.

Q:4

What is charachteristic equation? Page no 379

Answer:

$$\det(A-\lambda I)=0$$

This equation is called the characteristic equation of the matrix A.

Q:5

Can we extend power series?

AnsweR: Page no 268

I answered in yes and then wrote the extended form of power series.

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots.$$

Q:6 Page no 371

Find the derivative and the integral of the following matrix

$$X(t)=\begin{pmatrix} \sin 2t \\ e^{3t} \\ 8t-1 \end{pmatrix}$$

Solution:

The derivative and integral of the given matrix are, respectively, given by

$$X'(t)=\begin{pmatrix} \frac{d}{dt}(\sin 2t) \\ \frac{d}{dt}(e^{3t}) \\ \frac{d}{dt}(8t-1) \end{pmatrix}=\begin{pmatrix} 2\cos 2t \\ 3e^{3t} \\ 8 \end{pmatrix}$$

Q:7

Write system of equation in matrix form?

Solution: Page no 387

$$\frac{dx}{dt} = -3x + 4y - 9z$$
$$\frac{dy}{dt} = 6x - y$$
$$\frac{dz}{dt} = 10x + 4y + 3z$$
Solution :
$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Q:8 Page no 98

Dudce special case of logistic equation (epidemic spread)? (5)

The natural assumption is that the rate $\frac{dx}{dt}$ of spread of disease is proportional to the number $x(t)$ of the infected people and number $y(t)$ of people not infected people. Then

$$\boxed{\frac{dx}{dt} = kxy}$$

Since $\boxed{x + y = n + 1}$

Therefore, we have the following initial value problem

$$\frac{dx}{dt} = kx(n+1-x), \qquad x(0) = 1$$

The last equation is a **special case of the logistic equation** and has also been used for the **spread of information** and the **impact of advertising** in centers of population.

Q:9

Find order of homogenous equation obtained from non homogenous differential equation:

$y'' + 4y' + 3y = 4x^2 + 5$? **? (2 MARKS)**

Q:10:

Find a series solution for the differential equation $y''+y=0$ about $x_0=0$ such that

Find condition of cofficent for $a_{n+2} \& a_n \, (c_{n+2} \& c_n)$?

Q:11

Which series is identically zero? Page no 273

Answer:

Series that are Identically Zero

If for all real numbers x in the interval of convergence, a power series is identically zero i.e.

$$\sum_{n=o}^{\infty} c_n (x-a)^n = 0, \qquad R>0$$

Then all the coefficients in the power series are zero. Thus we can write

$$c_n = 0, \qquad \forall \ n = 0, 1, 2, \ldots$$

Q:12

$A \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix}$

eigen values ?

Eigen vectors ?

Note: I am not going to solve this question solve it by your self by consulting two examples below.

=======================First Paper End=====================

Q1: Find Coefficient of metrix:

$$\frac{dx}{dt} = -3x - 2y$$

$$\frac{dy}{dt} = 5x + 7y$$

Solution:

Cofficent of matrix =

$$A \begin{bmatrix} -3 & -2 \\ 5 & 7 \end{bmatrix}$$

Q2: Eigen Values of metrics.

A=$\begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}$

Consider the question below:

$$A \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$$

$$(3-\lambda)(-3-\lambda) + 36 = 0$$

$$3(-3-\lambda) - \lambda(-3-\lambda) + 36 = 0$$

$$-9 - 3\lambda + 3\lambda + \lambda^2 + 36 = 0$$

$$-9 + \lambda^2 + 36 = 0$$

$$\lambda^2 + 27 = 0$$

$\lambda = \sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

This question is similar to above.

Q3: whether or not a singular points have real number if not then give some examples?

Answer: Page no 284

(b) The singular points need not be real numbers.
The equation $(x^2+1)y''+2xy'+6y=0$ has the singular points at the solutions of $x^2+1=0$, namely, $x=\pm i$.

Q4: Solve the differential equation. $\frac{1}{y}\frac{dy}{dx} \quad 1$

Solution:

$$\frac{1}{y}\frac{dy}{dx} \quad 1$$

$$\frac{dy}{y} \quad (1)dx$$

$$\int\frac{dy}{y} \quad \int(1)dx$$

$$\ln y = x + c$$

$$y \quad e^{x+c}$$

Q5: complementary solution of DE

$$y'' - 4y' + 4y = 2e^{2x}$$

Solution: Page no 182

Step 2 To find the complementary function, we consider

$$y'' - 2y' + y = 0$$

The auxiliary equation is

$$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0 \Rightarrow m = 1, 1$$

The complementary function for the given equation is

$$y_c = c_1 e^x + c_2 x e^x$$

Q6: state the Bessel's function of first kind of order ½ and -1/2.

Solution: Page no 313

$$J_v(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)\Gamma(1+v+n)} \left(\frac{x}{2}\right)^{2n+v} \quad (6)$$

Also for the second exponent $r_2 = -v$, we have

$$J_{-v}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)\Gamma(1-v+n)} \left(\frac{x}{2}\right)^{2n-v} \quad (7)$$

Only put the value of ½ in J_v (x) and - ½ in J_{-v} (x) at the places of v.

Q7: Define the derivative of

$$A(t) = \begin{bmatrix} e^{2t} \\ t^2 \\ 8 \end{bmatrix}$$

Answer: Repeated

Q8: Find the egien values of

$$A \begin{bmatrix} 1 & -1 \\ \frac{4}{9} & \frac{-1}{3} \end{bmatrix}$$

Solution:

Consider the question below.

$A \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$

for eigen values, $|A - \lambda I| = 0$

$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$

$(3-\lambda)(-3-\lambda)+36=0$

$3(-3-\lambda)-\lambda(-3-\lambda)+36=0$

$-9-3\lambda+3\lambda+\lambda^2+36=0$

$-9+\lambda^2+36=0$

$\lambda^2+27=0$

$\lambda=\sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

Q9: bht lamba tha mery sy note ni hoa time thora tha is lia ☹

Q10: Find the auxiliary solution of $x^t = 3x - y - 1$ and $y^t = y + x - 4e^t$

Consult page no 141

Q11: Write down the system of differential equations (5marks)

$\frac{dx}{dt} = 6x + y + 6t$, $\frac{dy}{dt} = 4x + 3y - 10t + 4$

In form of $X' = AX + F(t)$

Solution:

$X' = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} X + \begin{pmatrix} 6t \\ -10t+4 \end{pmatrix}$

====================== PAST PAPERS===========================

Q: An electronic component of an electronic circuit that has the ability to store charge and opposes any change of voltage in the circuit is called

Inductor
Resistor
Capacitor
None of them

Q: If A_o is initial value and T denotes the half-life of the radioactive substance than

$T \quad \frac{1}{2A}$

$\frac{dA}{dt} \quad KA$

$A(T) \quad \frac{A_0}{2}$

None of the above

Q: integrating factor of the given equation $x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)$ is

Xsecx

Cosx

Cotx

Xsinx

Q: Operator method is the method of the solution of a system of linear homogeneous or linear non-homogeneous differential equations which is based on the process of systematic elimination of the
Dependent variables

Independent variable
Choice variable
None of them

Q: If E (t) =0, R =0 Electric vibration of the circuit is called__________

Free damped oscillation
Un- damped oscillation
Over damped oscillation
None of the given

Q: Eigen value of a matrix $\begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$

5, 5

10, 5

25, 5

None

Q: Eigen value of a matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

2,0

1,1

1,2

None

Q: For Eigen values λ 5,5 of a matrix $A \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$,there exists......... Eigen vectors.

infinite
one
two
three

Q: If a matrix has 1 row and 3columns then the given matrix is called________

Column matrix

Row matrix

Rectangular matrix

None

Q: The general solution of differential equation $\frac{dy}{dx} \quad \frac{x+y}{x}$ .is given by

$e^{\frac{y}{x}} \quad cx$

$e^{\frac{y}{x}} \quad cy$

$e^{\frac{x}{y}} \quad cx$

$e^{-\frac{x}{y}} \quad cx$

Q: The integrating factor of the D.E $\frac{dy}{dx} + y\ln y = ye^{x}$ is

e^{x}

e^{y}

$e^{\frac{1}{x}}$

$e^{\frac{x}{y}}$

Q: For the equation of free damped motion $\frac{dx^2}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = 0$ the roots are

$m_1 = -\lambda + \sqrt{\lambda^2 + \omega^2}$ & $m_1 = -\lambda - \sqrt{\lambda^2 + \omega^2}$ if$\lambda^2 - \omega^2 > 0$ Then the equations said to be:

Under damped
Over damped
Critically damped
None of them

Q: For the system of differential equations $\frac{dy}{dt}=2x, \frac{dx}{dt}=3y$ the independent variable is **(Are)**

X,t

Y,t

X,y

t

Q: For the system of differential equations $\frac{dy}{dt}=2x, \frac{dx}{dt}=3y$ the dependent variable is **(Are)**

X,t

Y,t

X,y

t

Q: $\begin{pmatrix} 4-\lambda & 1 & 0 \\ 0 & 4-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0$ gives

λ 4 of multiplicity of 1

λ 4 of multiplicity of 2

λ 4 of multiplicity of 3

None of the given.

Q: wronksin of x, x^2 is

x^2

x

0

None of the above

a) Matrix A nd value of lembda was given to find the eigen vector? 3 marks.

Answer: (This question is solved by Shining Star as original question was missing so I put it here for reference.)

A= $\begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}$, corresponding Eigen value $\lambda = -2$.

$$\left(\begin{array}{cc|c} -3-(-2) & 1 & 0 \\ 2 & -4-(-2) & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 2 & -2 & 0 \end{array}\right)$$

Add two times row 1 in row 2

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$-k_1 + k_2 = 0$

$k_1 \quad k_2$

Choosing $k_2 = 1$, we get $k_1 = 1$

therefore, eigen vector is $V \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) X'=AX was given to find the eigenvalue and Eigen vector? 5 marks. (This question is solved by Shining Star as original question was missing so I put it here for reference.)

For eigen values consut this question and for eigen vector look at the above.

$$X' \quad \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

Solution:

$$X' \quad \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

$$A \quad \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$$

$$(3-\lambda)(-3-\lambda)+36=0$$

$$3(-3-\lambda)-\lambda(-3-\lambda)+36=0$$

$$-9-3\lambda+3\lambda+\lambda^2+36=0$$

$$-9+\lambda^2+36=0$$

$$\lambda^2+27=0$$

$\lambda = \sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

c) Solve DE dy-7dx=0 for initial value f(0)=1? 5 marks.

Answer:

$dy - 7dx = 0$

$dy \quad 7dx$

$\int dy \quad \int 7dx$

$y = 7(x) + c$

$f(0) \quad 1$

$f(0) = 7(0) + c$

$f(0) = 0 + c$

$1 \quad C$

$y = 7x + 1$

d) Find the general solution of 4x^2 y '' + 4xy' (4x^2-25)y=0 (it is the Bessel's Equation a nd same question is given in exercise pg 314 of our handouts)? 5 marks

Answer:

Bessel's differential equation is

$$x^2 y'' + xy' + \left(x^2 - v^2\right) y = 0$$

Example 1

Find the general solution of the equation

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0 \text{ on } (0, \infty)$$

Solution

The Bessel differential equation is

$$x^2 y'' + xy' + \left(x^2 - v^2\right) y = 0 \qquad (1)$$

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0 \qquad (2)$$

Comparing (1) and (2), we get $v^2 = \frac{1}{4}$, therefore $v = \pm\frac{1}{2}$

So general solution of (1) is $y = C_1 J_{1/2}(x) + C_2 J_{-1/2}(x)$

Answer:

e) When a function is said to be analytic at any point? 2 marks
Answer:
A function is said to be analytic at point if the function can be represented by power
series in (x−a) with a positive radius of convergence.

f) What is the ratio test? (its on pg 264 of our handouts) 5 marks

To determine for which values of x a power series is convergent, one can often use the Ratio Test. The Ratio test states that if

$$\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} c_n (x-a)^n$$

is a power series and

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right| |x-a| = L$$

Then:

- The power series converges absolutely for those values of x for which $L<1$.
- The power series diverges for those values of $x$for which $L>1$ or $L = \infty$.
- The test is inconclusive for those values of x for which $L=1$.

g) What is the formula for radius of convergence? (Its on pg 265 of our hndouts)2 marks

Answer:

$$R = \lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|$$

h) Write system of linear differential equations for two variables x and y? (its on pg 333 of our handouts).2 marks
i) write any 3 D.Es of order 2? 3 marks Page no 207
Answer:

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$$

j) D.E was given to convert in normal form? 3 marks
Answer:

Reduce the third-order equation

$$2y''' = -y - 4y' + 6y'' + \sin t$$

or

$$2y''' - 6y'' + 4y' + y = \sin t$$

to the normal form.

Solution: Write the differential equation as

$$y''' = -\frac{1}{2}y - 2y' + 3y'' + \frac{1}{2}\sin t$$

Now introduce the variables

$$y = x_1,\ y' = x_2,\ y'' = x_3.$$

Then

$$x_1' = y' = x_2$$
$$x_2' = y'' = x_3$$
$$x_3' = y'''$$

Hence, we can write the given differential equation in the linear normal form

$$x_1' = x_2$$
$$x_2' = x_3$$
$$x_3' = -\frac{1}{2}x_1 - 2x_2 + 3x_3 + \frac{1}{2}\sin t$$

k) Any example of boundary value problem? 2 marks

Consider the function

$$y = 3x^2 - 6x + 3$$

We can prove that this function is a solution of the boundary-value problem

$$x^2 y'' - 2xy' + 2y = 6,$$
$$y(1) = 0, \qquad y(2) = 3$$

Since $\quad \dfrac{dy}{dx} = 6x - 6, \ \dfrac{d^2y}{dx^2} = 6$

Therefore $\quad x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = 6x^2 - 12x^2 + 12x + 6x^2 - 12x + 6 = 6$

Also $\quad y(1) = 3 - 6 + 3 = 0, \quad y(2) = 12 - 12 + 3 = 3$

Therefore, the function ' y satisfies both the differential equation and the boundary conditions. Hence y is a solution of the boundary value problem.

Note: Power series sy ziada NHI tha. Lecture 35 to 45 pr ziada emphsis tha

Q No.2 ------------------5 marks:

Write annihilator operator for x+3xe^ (6x) e ki power 6 xs

$$g(x) = 4e^{2x} - 6xe^{2x}$$

$$(D-2)^2\left(4e^{2x} - 6xe^{2x}\right) = (D^2 - 4D + 4)(4e^{2x}) - (D^2 - 4D + 4)(6xe^{2x})$$

or $\quad (D-2)^2\left(4e^{2x} - 6xe^{2x}\right) = 32e^{2x} - 32e^{2x} + 48xe^{2x} - 48xe^{2x} + 24e^{2x} - 24e^{2x}$

or $\quad (D-2)^2\left(4e^{2x} - 6xe^{2x}\right) = 0$

Therefore, the annihilator operator of the function g is given by

$$L = (D-2)^2$$

We notice that in this case $\alpha = 2 = n$.

Q No.3 ------------------3 marks:

Write the solution of simple harmonic motion in alternative simpler form x(t)=c1coswt+c2sinwt from lec 22 page 199

Answer:

Q No.4 ------------------2 marks:

Define general linear DE of nth order

Answer:

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}}+\cdots+a_1(x)\frac{dy}{dx}+a_0(x)y=g(x)$$

Define elementary row operation.

Answer:

Addition or multiplication of two rows.

The elementary row operations consist of the following three operations

- Multiply a row by a non-zero constant.
- Interchange any row with another row.
- Add a non-zero constant multiple of one row to another row.

Eigenvalue of multiplicity m 3

Answer:

Suppose that m is a positive integer and $(\lambda - \lambda_1)^m$ is a factor of the characteristic equation

$$\det(A - \lambda I) = 0$$

Further, suppose that $(\lambda - \lambda_1)^{m+1}$ is not a factor of the characteristic equation. Then the number λ_1 is said to be an eigenvalue of the coefficient matrix of multiplicity m .

Fundamental of matrix 3

Answer:

Suppose that the a fundamental set of n solution vectors of a homogeneous system $X' = AX$, on an interval I , consists of the vectors

$$X_1 = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{pmatrix}, X_2 = \begin{pmatrix} x_{12} \\ x_{22} \\ \vdots \\ x_{n2} \end{pmatrix}, \ldots, X_n = \begin{pmatrix} x_{1n} \\ x_{2n} \\ \vdots \\ x_{nn} \end{pmatrix}$$

Then a fundamental matrix of the system on the interval I is given by

$$\phi(t) = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{pmatrix}$$

What is determinnant? How to find it.

Determinant of a Matrix

Associated with every square matrix A of constants, there is a number called the determinant of the matrix, which is denoted by $\det(A)$ or $|A|$

Write equation in matrix form.

Find general solution........ 5marks..

Forbenius Theorem.........

$$y = (x - x_0)^r \sum_{n=0}^{\infty} c_n (x - x_0)^n = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r}$$

5marks

Super position method for vectors

Answer:

$$y = C_1 y_1(x) + C_2 y_2(x)$$

Explain convergence and infinty condition of a infintye sereies.

- If we choose a specified value of the variable x then the power series becomes an infinite series of constants. If, for the given x, the sum of terms of the power series equals a finite real number, then the series is said to be convergent at x.

What does these symbols mean?

Symbol	Meaning
R_{ij}	Interchange the rows i and j.
cR_i	Multiply the ith row by a nonzero constant c.
$cR_i + R_j$	Multiply the ith row by c and then add to the jth row.

Q2. **Solve the system of differential equations** $\frac{dy}{dt} = x, \frac{dx}{dt} = y$ **by systematic elimination.**

Solution:

$\frac{dy}{dt} = x \Rightarrow Dy - x = 0 \qquad ....(i)$

$\frac{dx}{dt} = y \Rightarrow -y + Dx = 0 \qquad ....(ii)$

Operate (ii) by D, we get

$-Dy + D^2x = 0$ ....(iii)

Add (i) and (iii), we get

$Dy - x = 0$

$\underline{-Dy + D^2x = 0}$

$D^2x - x = 0$

$\left(D^2 - 1\right)x = 0$

Auxiliary equation is $m^2 - 1 = 0$

$m = \pm 1$

$\boxed{x(t) \quad c_1e^t + c_2e^{-t}}$

Put this in (i), we get

$Dy - \left[c_1e^t + c_2e^{-t}\right] = 0$

$Dy = c_1e^t + c_2e^{-t}$

Integrate both sides, we get

$\boxed{y(t) = c_1e^t - c_2e^{-t}}$

Q3. **Find a series solution for the differential equation** $y'' + y = 0$ **about** $x_0 = 0$ **such that**

$a_{n+2} = -\frac{a_n}{(n+2)(n+1)} \qquad n = 0, 1, 2, \ldots \qquad$ **and** $\qquad y(x) \quad \sum_{n \ 0}^{\infty} a_n x^n$

Solution:

$$a_{n+2} = -\frac{a_n}{(n+2)(n+1)} \;; \qquad n = 0,1,2,\ldots$$

For $n=0,\ a_2 = -\frac{a_0}{(0+2)(0+1)} = -\frac{a_0}{2}$

For $n=1,\ a_3 = -\frac{a_1}{(1+2)(1+1)} = -\frac{a_1}{6}$

For $n=2,\ a_4 = -\frac{a_2}{(2+2)(2+1)} = -\frac{a_2}{12} = -\frac{1}{12}\left(-\frac{a_0}{2}\right) = \frac{a_0}{24}$

For $n=3,\ a_5 = -\frac{a_3}{(3+2)(3+1)} = -\frac{a_3}{20} = -\frac{1}{20}\left(-\frac{a_1}{6}\right) = \frac{a_1}{120}$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \ldots$$

$$y(x) = a_0 + a_1x + \left(-\frac{a_0}{2}\right)x^2 + \left(-\frac{a_1}{6}\right)x^3 + \left(\frac{a_0}{24}\right)x^4 + \left(\frac{a_1}{120}\right)x^5 + \ldots$$

$$y(x) = a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \ldots\right) + a_1\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \ldots\right)$$

Q4. **Write solution** $X = \frac{4}{3}Cos3t - \frac{5}{3}Sin3t$ **in the form** $X = A.Sin(wt+\phi)$**.**

$$A = \sqrt{\left(\frac{4}{3}\right)^2 + \left(-\frac{5}{3}\right)^2} = \frac{\sqrt{41}}{3}$$

$$\phi = \tan^{-1}\left(\frac{4/3}{-5/3}\right) = 0.6747 \text{ radians}$$

$$x(t) = \frac{\sqrt{41}}{3}\sin(3t + 0.6747)$$

Q5. If the equation $M(x,y)dx + N(x,y)dy = 0$ is not exact,

Case 1:

When $\exists$ an integrating factor $u(y)$, a function of y only. This happens if the expression

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$$

is a function of y

Case2:

If the given equation is homogeneous and

$$xM + yN \neq 0$$

Then find the integrating factor in both cases.

Solution:

$$u \quad \frac{1}{xM + yN}$$

Q8. **Under which conditions linear independence of the solutions for the differential equation** $y'' + P(x)y' + Q(x)y = 0$ ....(1) **is guaranteed?**

Solution:

Linear independence is guaranteed in case when the Wronskian of the two solutions is not equal to zero.

Q10. **When Frobenius' Theorem is used in Differential Equation** $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$ **?**

Solution:

When we have a regular singular point x= x0, then we can find at least one series solution of the form $y = \sum_{n \; 0}^{\infty} c_n \left(x - x_0\right)^{n+r}$, where r is the constant that we will determine after solving the differential equation.

Q12. Define Legendre's polynomial of degree n

Solution:

Legendre polynomial is an n^{th} degree polynomial and it is given by the formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$$

Q13. **What is the ordinary differential equation and give an example?**

Solution:

A differential equation which only includes ordinary derivatives is known as ordinary differential equation. Some examples of ordinary differential equations include:

$$\frac{dy}{dx} = x^2 + y$$

$$\frac{dy}{dx} = \left(x^2 + 1\right)\left(y^2 + 1\right)$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 0$$