① 22/05

Lec# 01

Mth601 . Operation Research

Introduction;

"OR is concerned with scientifically deciding How to best design & operate Man-Machine System, usually under conditions requiring the allocation of scare resources."

* Management: decision making & control.
* Management science; study of problems as abstractions and the application of the resulting theory to practical situations.

=> Two Fundamental disciplines;

(i) Behavioral science

(ii) Science and Quantitative methods.

* Operation Research is the application of Quantitative Method to decision Making.

OR Approach to problem solving.

① Observation ② Define a problem

③ Model construction ④ Solution

⑤ Implementation.

Lec# 02 OR Techniques;

1. Linear Mathematical programming techniqu
2. Probabilistic techniques.
3. Inventory techniques.
4. Network techniques.
5. others (Non-linear programming)

Uses of OR;

* Macro-Economic planning;
  - (a) Input & output Analysis
  - (b) investment planning
  - (c) Choice of project;
  - (d) OR can be used in simulation Modeling of the Economay of the Countary,

* Sectorial planning.

OR can be employed in a particular sector of the Economy. in agricture, in Finance, in industry, in production etc.

* Micro Economic planning:

This sorts of activity involve planning the operations of a Company. Improving the layout of a workshop in a Company.

Lecture # 04

Network Analysis (PERT/CPM)

* PERT stand for "program Evaluation and Review Technique". (Event-oriented technique)
* CPM stand for "Critical path Method."

=) PERT is useful in research and developmental projects.

=) CPM is mostly used in construction projects, or in situtions already Handled.

=) If A projected is completed in reduced duration it's Known As (Crash duration).

(Concept of Network)

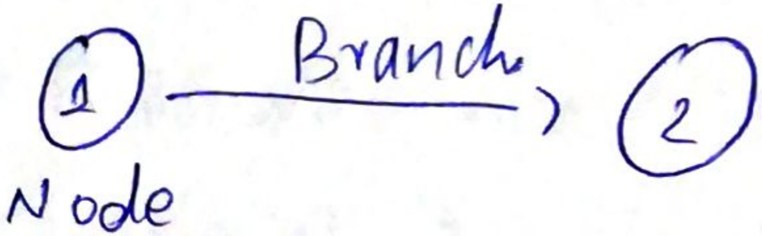
=) A Network is defined as a graphic representation with a flow of some type in its branches. It represent Nodes and branches.

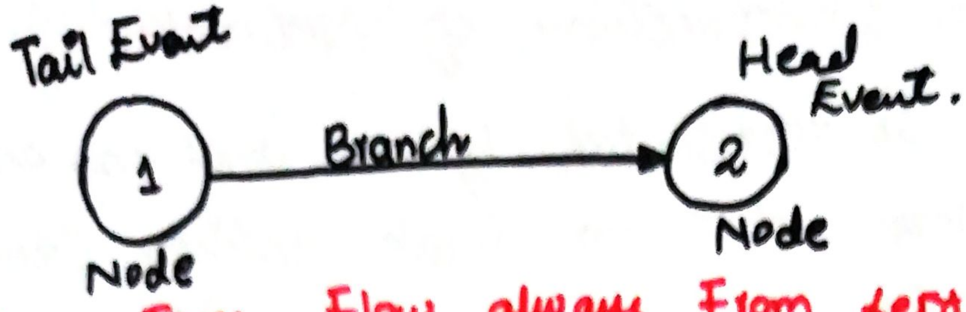
=) Node is the intersection of two branch lines. Denoted by ○ circle.

=) Each Branch repsents an activity.

=) Each Node represents an event.

=> The arrowheads indicate the sequence in which event must be achieved.



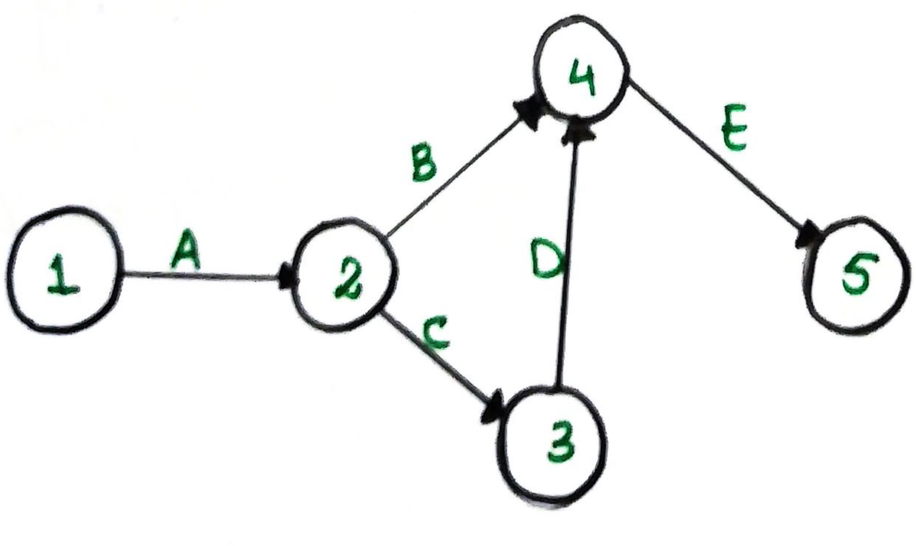


Note:- Time Flow always From left to right, so value of Head Event Must be greater than Tail Event.

Example 01:- A Company is Intersted in preparing a budget. The details of the activites and the departments involved are given in the Table:-

The project of preparation of a production budget.

Job Identification	Alternate	Job Description	Department.
A	1-2	Forecasting sales	Sales
B	2-4	Pricing sales	Sales
C	2-3	Preparing production sech	Engineering
D	3-4	Costing the production	Costing
E	4-5	preparation of budget	preasident.



* Rules For Construction of Network:-

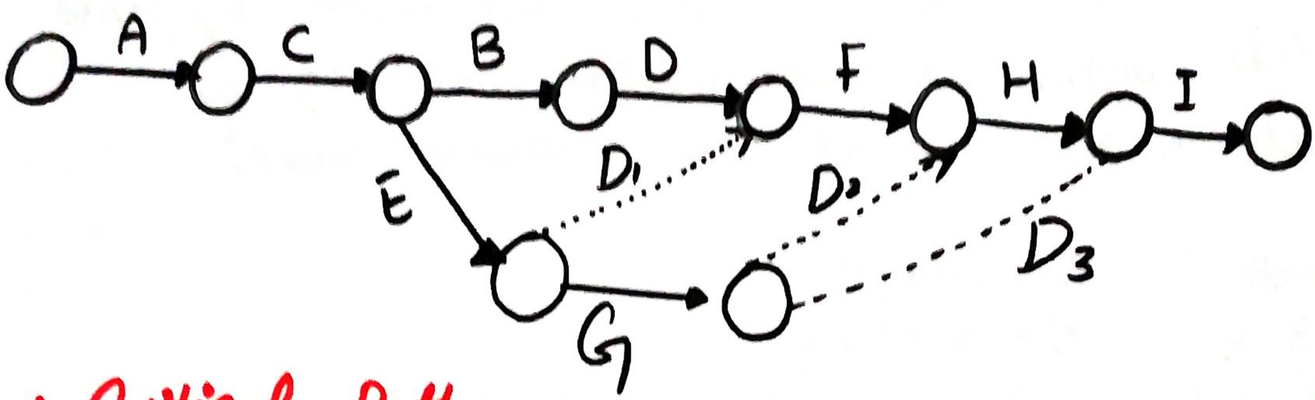
1. Each activity is represented by one and only one arrow. This Means that no single activity Can be represented twice in a Network.
2. Now two Activity Can be Identified by the Same end events. This Means that there Should not be loops in the Network.
3. Time follows From left to Right. All the arrows points in one direction.
4. Arrows Should not Cross each other.
5. Every node Must have at least one activity preceding it and at least one activity Following it, except For the nodes at the very beginning and at the very last end of the Network.

* Dummy Activity:-

There is a Need of dummy activities when the project Contains groups of two or More jobs which have common predecessors. The time taken for the dummy activities is zero.

Example:-
A project Consists of the following activities whose precedence relationship is given blew. Draw an Arrow diagram to represent the project.

Activity	Followed by	Preceded by
A	B, C	–
B	D	C, A
C	E, B	A
D	F	B
E	G, F	C
F	H	E, D
G	H, I	E
H	I	G, F
I	–	G, H.

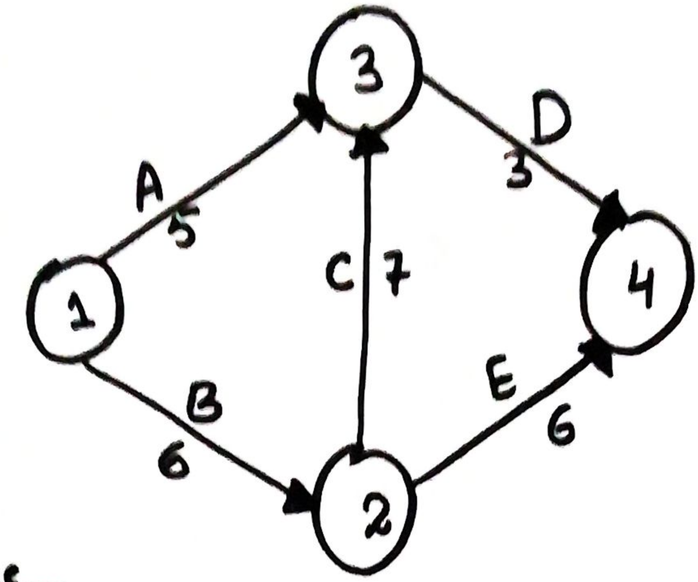


★ Critical Path;

After listing all the activities with their precedence relationship, we have to Find the Minimum time required For completion of the entire project. For this we Must Find the longest path with the Sequences of Connected activities, through the Network. This is called the **Critical path** of the Network and its length determine the time for completion of the project.

Examples:-

Consider the Following Network:-



Path:- "The set of nodes connected by lines which begin at the initial Node of a Network and end at the terminal Node."

Path	Time (days)
1-3-4	5+ 3 = 8 days
1-2-3-4	6+ 7+3 = 16
1-2-4	6 +6 = 12

The path connecting the Nodes 1, 2, 3 and 4. Constitutes the longest path and Hence 1-2-3-4 is the Critical path.

The Minimum time to Complete the project is the time taken for the longest path namely 16 days.

* Some More Basic Rules & Concepts;

1. Looping & Dangling is not allowed.
2. Merge Node:- (Many to one)
3. Burst Node:- (one to Many)

* Time Activities:-

① Earliest start time:-

Earliest possible time at which an activity can start and is given by the earliest time of the tail Event.

② Earliest Finish time:-

Earliest possible time at which an activity can Finish and is given by adding the duration time to the earliest start time.

③ Latest Finish time:-

Latest Event time of the Head Event.

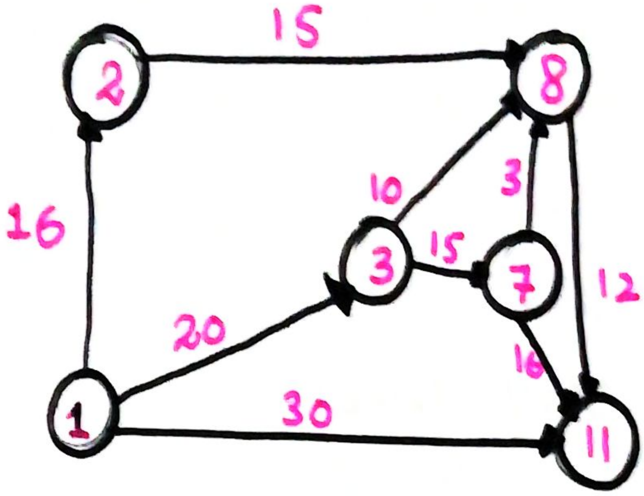
④ Latest start time:-

Latest possible time by which an activity Start and is given by Subtracting the duration time from the Latest Finish time.

Notations:-

Earliest time of Tail Event = i_E
Latest Time of tail Event = i_L
Earliest time of Head Event = j_E
Latest time of Head Event = j_L

Example of Time Event.



1 = 0

2 = 16

3 = 20

7 = 20+15 = 35

8 = 16+15 OR 20+10 OR [20+15+3] = 38

11 = 30 OR [35+16] OR 38+12 = 51.

★ Slack or Float:-

(i) Independent Float:-

Time By which an activity can expand without affecting others activities. It is computed by subtracting the tail event slack from the Free Float.

$$I = j_E - j_L - D$$

If Negative:- Take $I = 0$.

(ii) Free Float:-

Time taken by which an activity expand without affecting subsequent activity.

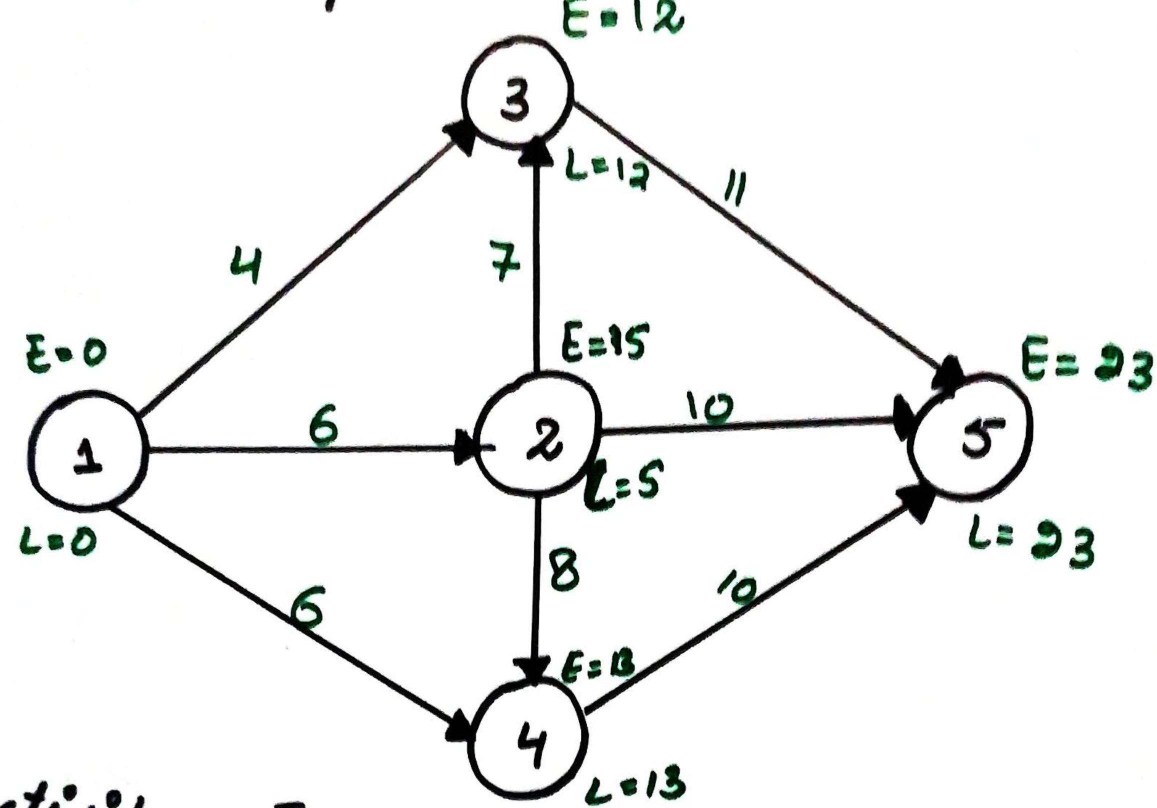
$$F = j_E - i_E - D$$

Total Float:-

Time Taken by which an activity can expand without affecting the overall duration of the project.

$$T = j_L - i_E - D$$

Example:-



Activity For 1-3:-

- $T = j_L - j_E - D = 12 - 0 - 4 = 8$
- $F = j_E - i_E - D = 12 - 0 - 4 = 8$
- $I = j_E - i_L - D = 12 - 0 - 4 = 8$

For your Exercise;

1-2 = ? , 1-4, = ? 2-3, 2-4, 2-5, 3-5, 4,5=.

Critical path;

1,-2,-3,-5,

1,2,-4,-5

~~#08~~

(i) Expected length of a critical path;

The expected length of a sequence of independent activities is simply the sum of their separate expected lengths.

=> T_e is the sum of t_e's along the critical path, then variance of T_e equals the sum of all the variance of the critical activites.

=> The standard deviation of the expected project duration is the square root of the variance T_e as calculated before.

=> Formula for finding Expected ~~long~~ time

$$t_e = \frac{t_o + 4t_m + t_p}{6} \Rightarrow$$

$$t_e = \frac{1+4+7}{6} = 2$$

=) probability of completing a proj[illegible] with given data.

* It is known that in a normal distribution the Area under the normal curve gives the probability.

For $Z = 0$, $T = T_e$.

* The probability of completing the project is 50% as represented by the Area to the left of the Central line.

$$Z = 0, \quad Z = \left[\frac{T - T_e}{\sigma}\right]$$

Example;

(The Following table lists the Jobs of a Network with their time estimate)

(See Table and Remaining Question From Handouts) (Page 31)

Solution;

we Have to Find

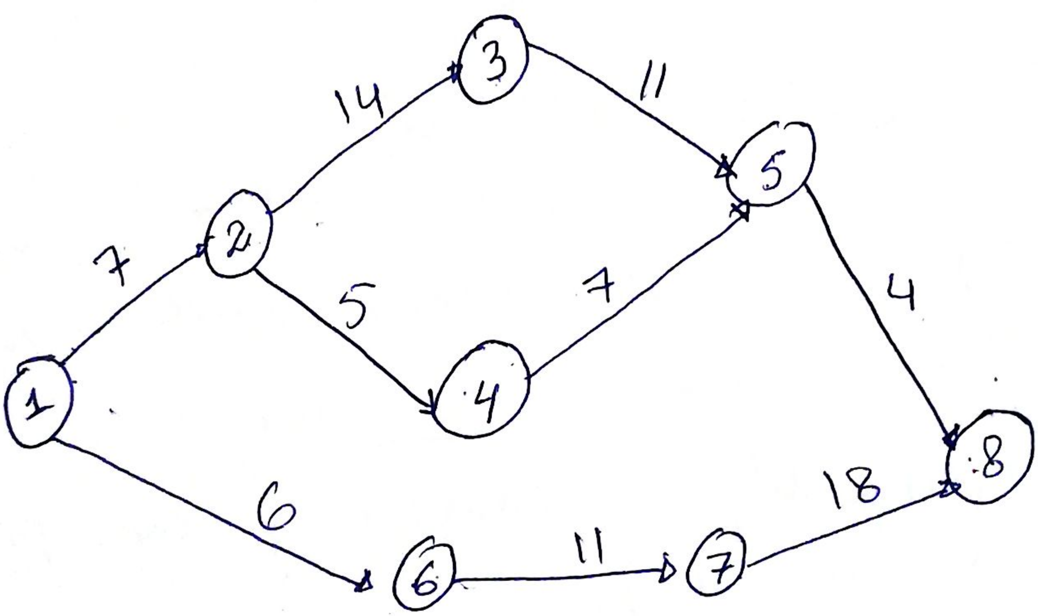
$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$S.D = \left(\frac{T_p - t_o}{6}\right)$$

$$\text{Variance} = \left(\frac{t_p - t_o}{6}\right)^2$$

Activity	t_e (Days)	S.D (days)	Variance
1-2	$t_e = \frac{t_o + 4t_m + t_p}{6} = 7$	$\frac{t_o - t_p}{6} = 2$	$\left(\frac{t_p - t_o}{6}\right)^2 = 4$
1-6	6	2	4
2-3	14	4	16
2-4	5	1	1
3-5	11	2	4
4-5	7	2	4
6-7	11	4	16
5-8	4	1	1
7-8	18	4	16

(a) Project Network;



(b) There are three paths.

1 - 2 - 3 - 5 - 8 = 7 + 14 + 11 + 4 = 36

1 - 2 - 4 - 5 - 8 = 7 + 5 + 7 + 4 = 23

1 - 6 - 7 - 8 = 6 + 11 + 18 = 35 days.

1, 2 - 3 - 5 - 8 is the longest Path. and Hence the Critical Path.

Expected length is 36 day.

Variance for

1 - 2	4
2 - 3	16
3 - 5	4
5 - 8	1

=) $4 + 16 + 4 + 1 = 25$
variance of projection duration is 25.

Standard deviation of the project duration $= \sigma = \sqrt{V} = \sqrt{25} = 5$ days.

(c) due date = 42 days

Expected duration = 36 days (T_e) and S.D = 5 days

$$Z = (T - T_e) / S.D$$
$$= (42 - 36)/5 = 1.2$$

The Area under noromal curve for $Z = 1.2$ is 0.3849

The probability of completing the projection in 42 days.

$= 0.5000 + 0.3849$

$= 0.8849$

$= 88.49\%$

1) Resource Secheduling in Network.

* A Heuristic can be defined as a "guide or Method of reducing search in problem solving situations".

2) A Number of different Heuristic programmes for scheduling projects with limited resources have Been developed recenty. These programmes are calssified in any of the two;

1. Resource leveling programmes;
2. Resource allocation Programmes.

Lecture # 11

(Segment 3 Inventory Control)
(Lec# 11 to 16)

1- Inventory Control;

=> Inventory is the Physical stock of item held in any business for the purpose of future production or sale.

In a production shop the Inventory may be in the form of raw Materials.

we shall be dealing only with the Finished goods inventory.

=> Inventory planning is the determination of the type and quantity of Inventory item that would be required at Future point for maintaining production Schedules

=> In Inventory control, we are primarily Concerned with the Inventory cost Control.

=> Two important Questions are;

① How Much to stock or How Much to buy

② How often to buy or when to buy.

=> Answers to the above Questions is given by certain "Mathematical Models, "Economic Order Quantity Models" (E.O.Q)

=) Inventory costs;

① Item cost, Rs C_1/item.
② ordering cost, Rs. C_2/order
③ Holding Cost Rs. C_3/item/unit time
④ Shortage Cost Rs. C_4/item/unit time

=) Item Cost (C_1)/item

This is the cost of the item whether it is Manufactured or purched.

=) Ordering Cost (C_2)/order.

When a unit is Manufactured, there unit setup cost (ordering cost) include the cost of Labour and Materials used in the setup and setup testing & Training cost.

=) Holding Cost (C_3)/item/unit time.

If the item is Held in stock, the cost involved in the ~~item~~ item carrying or Holding Cost. Some of the costs are,

① Taxes on inventories.
② cost of Maintaining inventory Record

* Unit of time may be days, Months, weeks or years.

⇒ Shortage Cost (C_u)/per item per unit time of shortage.

The shortage cost is due to the delay in ~~st~~ satisfying demand (due to wrong planning); but the demand is eventully satisfied after a period of time.

* The unit Shortage Cost includes such items as;

1. overtime requirements due to shortage.
2. Cost of expediting
3. loss of goodwill of customers due to dealy.
4. Lost production time.

(E.O.Q Models)

The inventory Control Model can be broadly Classified into two categories;

1. Determintic inventory problems.
2. Probabilistic inventory problems.

In the determintic; The parameter like demand, ordering quantity cost, etc are already known and there is no uncentainty.

* But in probabilistic, the uncertain aspects are taken into account.

There are Four EOQ Models which are discussed below;

(Mode - 1) Purchasing Model with no Shortages (Wilson's Model)

Assumptions ~~are~~ for deriving the Formula for economic order Quantity;

1. Demand is at a Constant rate.
2. Replacement of item is instantaneous (lead time is zero)
3. The Cost coefficients C_1, C_2 and C_3 are Constant.
4. There is no shortage cost or $C_4 = 0$

(This is also known as Saw tooth Model)

The Total cost for this Model for one cycle is Made up of three Components:-

- Total cost/period = (item cost + Setup cost + holding cost/period)

- Item cost per period = (cost of item) x (number of items ordered/period).

$$= C_1 Q \rightarrow (1)$$

- Purchase or setup cost per period = C_2 (only one setup per period).

- Item holding cost per period = (Holding cost) x (average inventory per period) x (time per period)

$$= C_3 Q/2 \times t \rightarrow (2)$$

Therefore the Total cost per period (C')

$$C' = C_1 Q + C_2 + C_3 Q/2 \times t \longrightarrow (3).$$

But the time for one period = $t = Q/D \rightarrow (4)$

$$C = C'/t \rightarrow (5)$$

$$\frac{C'}{t} = \frac{C_1 Q}{t} + \frac{C_2}{t} + C_3 Q/2 \, (t)/t$$

$$C = \frac{C_1 Q}{Q/D} + \frac{C_2}{Q/D} + C_3 Q/2$$

$$C = C_1 Q \times \frac{D}{Q} + C_2 \frac{D}{Q} + C_3 Q/2$$

$$C = C_1 D + C_2 D/Q + C_3 Q/2 \rightarrow (6)$$

Purchase cost = item Holding cost

$$C_2 = C_3 \times \frac{Q}{2} \times t$$

$$\frac{C_2}{t} = C_3 \times Q/2$$

$$\frac{C_2}{Q/D} = C_3 \times Q/2$$

$$2 C_2 D = C_3 Q^2$$

$$Q^2 = \frac{2 C_2 D}{C_3}$$

$$Q = \sqrt{\frac{2 C_2 D}{C_3}}$$

The value of Q is the economic order Quantity and any other order Quantity will result in a Higher Cost.

Corresponding period t is found from:

$$t = Q/D$$

Number of order is resiprocal of time.

6 (N= D/Q) we can determine optimum number of orders per year.

wher (D) is the Demand per year.

* (Model-2) (Purchasing Model with Shortages)

Formulas used;

$$Q^* = \sqrt{\frac{2C_2D}{C_3}} \cdot \sqrt{\frac{C_3+C_4}{C_4}}$$

$$S^* = \sqrt{\frac{2C_2D}{C_4}} \sqrt{\frac{C_3}{C_3+C_4}}$$

* (Model-3) (Manufacturing Model with no Shortages)

$$Q^* = \sqrt{\frac{2C_2D}{C_3}} \sqrt{\frac{R}{R-D}}$$

* Maximum Inventory;

$$I_m = Q(R-D)/R$$

* Time between orders

$$t = Q/D$$

* Time of Manufactures => $t_1 = Q/R$

* Optimum annual cost

Item cost + ordering cost + Holding cost

★ (Model - 4) Manufacturing Model with Shortages.

$$Q^* = \sqrt{\frac{2C_2D}{C_3(1-D/R)}}\sqrt{\frac{C_3+C_4}{C_4}}$$

$$S^* = \sqrt{\frac{2C_2D}{C_4}}\sqrt{1-D/R}\sqrt{\frac{C_3}{C_3+C_4}}$$

(Mth 601) Operation Research
Segment (IV) lecture # 22.
(Linear Programming)

(Simplex Methode)

Applications of Research Methods.

Ex; Maximize $Z = 3x + 5y$ subject to;
$x \leq 40$, $y \leq 30$, $x + y \leq 60$ & $x, y \geq 0$

Solution;

By Introducing slack variables S_1, S_2 & S_3 Convert the problem in Standard form;

$$Z = 3x + 5y + 0S_1 + 0S_2 + 0S_3$$

$$x + S_1 = 40$$
$$y + S_2 = 30$$
$$x + y + S_3 = 60$$

when $x = y = 0$

$$S_1 = 40; \quad S_2 = 30, \quad S_3 = 60;$$

Writting in Matrix Form; $AX = B$.

$$\begin{pmatrix} x & y & S_1 & S_2 & S_3 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} 40 \\ 30 \\ 60 \end{pmatrix}$$

		C_j	3	5	0	0	0	
C_B	B	x_B	x	y	S_1	S_2	S_3	Minimum Ratio x_B/y
0	S_1	40	1	0	1	0	0	40/0 = ∞
0	S_2	30	0	(1)	0	1	0	30/1 = 30 ←
0	S_3	60	1	1	0	0	1	60/1 = 60
$Z_j - C_j$			-3	-5 ↑	0	0	0	

Calculation for $Z_j - C_j$:-

$$Z_j - C_j = C_B X_j - C_j$$

$$Z_1 - C_1 = C_B X - C_1$$

$$(0 \times 1 + 0 \times 0 + 0 \times 1) - 3 = -3$$

$$Z_2 - C_2 = C_B y - C_2$$

$$= (0 \times 0 + 0 \times 1 + 0 \times 1) - 5 = -5$$

Some Important points to Remember;

① If all $(Z_j - C_j) \geq 0$ the optimal solution will obtained.

② if atleast one $(Z_j - C_j)$ is -ve then Indicate by an arrow and this Column is called Key Column.

③ If More than one $(Z_j - C_j)$ is -ve then Choose the Most Negative of them and Column is called the Key column.

Construct the New simplex table By Entering Incoming variables:- (Iteration 1)

		C_j	3	5	0	0	0	
C_B	B	x_B	x	y	S_1	S_2	S_3	Ratio
0	S_1	40	1	0	1	0	0	40
5	y	30	0	1	0	1	0	∞
0	S_3	30	1	0	0	-1	1	30
$z_j - C_j$			-3 ↑	0	0	5	0	

From Table;

$R_3 = 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 60$

$R_2 = 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 30$

$R_3 - R_2 = 1 \quad 0 \quad 0 \quad -1 \quad 1 \quad 30$

Calculations For $z_j - C_j$:-

$z_j - C_j = C_B X_j - C_j$

$z_1 - C_1 = C_B X - C_1$

$= (0 \times 1) - 3 = -3$

$z_2 - C_2 = C_B y - C_2$

$= (0 \times 0 + 5 \times 1 + 0 \times 0) - 5$

$= 5 - 5 = 0$

X is Inteing variable
S_3 is leaving variable;

(Iteration: 2)

		C_j	3	5	0	0	0	
C_B	B	X_B	x	y	s_1	s_2	s_3	Ratio
0	S_1	10	0	0	1	1	-1	
5	Y	30	0	1	0	1	0	
3	X	30	1	0	0	-1	1	
$Z_j - C_j$			0	0	0	2	3	

$R_1 = 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 40$

$R_3 = 1 \quad 0 \quad 0 \quad -1 \quad 1 \quad 30$

$R_1 - R_3 = 0 \quad 0 \quad 1 \quad 1 \quad -1 \quad 10$

Calculation For $Z_j - C_j$:-

$Z_j - C_j = C_B x_i - C_j$

$Z_1 - C_1 = C_B x - C_1$

$= (0 \times 0 + 5 \times 0 + 3 \times 1) - 3 = 3 - 3 = 0$

$Z_2 - C_2 = C_B y - C_2$

$(0 \times 0 + 5 \times 1 + 0 \times 3) - 5 = 5 - 5 = 0$

$Z_j - C_j \geq 0$

Now;

$x = 3, \; y = 5$ are optimal values

and Maximum $Z = 3x + 5y$

$= 3(3) + 5(5) \Rightarrow 9 + 25 = 34$

Maximum $Z = 34$.