

MTH632 MID TERM CURENT PAPERS

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Q1. Evaluate $\lim_{z \rightarrow \infty} \frac{z-1}{z^2+2+1}$

Q2 $\lim_{z \rightarrow i} \frac{z-1}{z^2+1}$

Q3 $\lim_{z \rightarrow 2+i} 2z^2 + 4 - 3z + 5i$

Q4. find $f'(y) \Rightarrow f(y) = (y + 6y^2)^3$

Q5. Find value of $(1 + \sqrt{3}i)^4$

Q6. Find $(1 + \sqrt{3}i)^{1/3}$ exponential form was given.

MCAQs. (i) z-w (2) Conjugate to
(3) reciprocal of z (4) ~~polar form of~~
(*) Almost paper was easy. MCAQs were too much easy.

Subjective MTH632

1. Show that polynomial $P(z) = 1 - z - z^2 + z^3 - z^4 + z^5$ is continuous at point $z = \frac{1}{2}$ in complex plane.

2. $\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{4 - x}$

3. $\lim_{z \rightarrow 2+i} (z^2 + 4z - 4 + 5i)$

4. $\lim_{z \rightarrow 3} (z^2 + 4z + 3)$

5. $f(z) = \frac{(1+z)^4}{z^2}$, find derivative, $f'(z)$.

6. Evaluate $\lim_{z \rightarrow 2i} (3z^2 - 4z + 6 - 6i)$

MR 632

① $(\sqrt{2}-i) - i(1-\sqrt{2}i) = -2i$ (2 marks)
Prove it

② $(1+i)^2 = 1+2i+2i^2$ prove it (2 marks)

③ Check Continuity of function at $z=i$ (3 marks)

$f(z) = \frac{z^2+1}{z^3+3z+2}$
Evaluate $\lim_{z \rightarrow 2+i} (z^2+2+5i-4z)$ (3 marks)

④ Find $f'(z)$ at $z=i$ (5 marks)

⑤ $f(z) = \frac{z}{(z+i)^3}$

evaluate

⑥ $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ (5 marks)

Mth632 2019 (Mids)

Q1:- $(-\sqrt{3}i)^2$ express it as $a+ib$ (2)

Q2:- evaluate $f(z) = \frac{z^2+3z+2}{z^2+1}$ at $z=i$ (2)

Q3:- $\lim_{z \rightarrow \infty} \frac{5z^2+1}{3z^2-z}$ (3)

Q4:- $\lim_{z \rightarrow 2+i} z^2-4z+2+5i$ (3)

Q5:- $f(z) = z^4 - 9z^2 + iz - 2$ check continuity at $z=-i$ (5)

Q6:- find $\lim_{n \rightarrow \infty} z_n$, $z_n = \frac{1}{n} \left(\cosh \frac{n}{3} + \sin n \left(\frac{\pi}{3} \right) i \right)$ (5)

$$\lim_{z \rightarrow 2+i} (z^2 - 4z + 2 + 5i)$$

$$\lim_{z \rightarrow 4} \frac{z - \sqrt{z}}{z - 2}$$

$$\lim_{z \rightarrow i} \text{of } f(z) = \frac{z-1}{z+1} \text{ at } z=i$$

- ① Verify that $(2, -3)(-2, 1) = (-1, 8)$ ②
- ② Test continuity of $f(z)$ at $z = i$ for

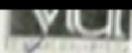
$$f(z) = \frac{z^2 + 3z + 2}{z^2 + 1}$$

- ③ Find $f'(z)$ if $f(z) = \frac{z-1}{2z+1}$ ③

- ④ Evaluate $\lim_{z \rightarrow 2+i} (z^2 - 4z + 2 + 5i)$ ③

- ⑤ Evaluate $\lim_{z \rightarrow 4} \frac{z - \sqrt{z}}{4 - z}$ ⑤

- ⑥ Find derivative of $f(z) = \frac{z-1}{z+1}$ at $z = i$ ⑤



FALL/SPRING MID/FINAL TERM 2018 (Year)

Rough Sheet

Campus Code: V81111

Date: 23/12/2018

Course Code: MTH 637

Superintendent Signature: _____

Student Signature: _____

Student ID: _____

~~23~~
① verify $(2, -3) \cdot (-2, 1) = (-1, 0)$

② $\lim_{z \rightarrow i} \frac{z-i}{z^2+1}$

③ $\lim_{z \rightarrow 4i} \frac{z^2+16}{z-4i}$

④ Find derivative of $f(z) = (iz^3 + 3z^2)^3$
at $z = 2i$

⑤ $\lim_{n \rightarrow \infty} z_n$ where $z = \frac{1}{n} \left(\cos\left(\frac{n\pi}{3}\right) + i \sin\left(\frac{n\pi}{3}\right) \right)$

⑥ Cauchy Riemann eqn in polar form

Show that $f'(z) = \frac{1}{2f(z)}$

Moreover

$$f(z) = \sqrt{z} e^{10/z}$$

Note: Submit this sheet to Superintendent, before leaving the Examination Center.



Rough Sheet

FALL/SPRING MID/FINAL TERM 2018 (Year)

Date: 11/11/2018

Supervisor Signature: _____

Student Signature: [Signature]

Computer Code: _____

Student ID: 1111111111

Course Code: 20711111

$(1-i)(1+i) = 1 - i^2 = 1 - (-1) = 2$

$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2} = \frac{1+2i+i^2}{2} = \frac{1+2i-1}{2} = \frac{2i}{2} = i$

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(1-i)^2}{1-i^2} = \frac{1-2i+i^2}{2} = \frac{1-2i-1}{2} = \frac{-2i}{2} = -i$

24) $\frac{1+2i}{3+4i} - \frac{1-i}{2+i}$

$= \frac{(1+2i)(2+i) - (1-i)(3+4i)}{(3+4i)(2+i)}$

$= \frac{2+5i+2i^2 - (3+4i-3i-4i^2)}{6+11i+8i^2}$

$= \frac{2+5i-2 - (3+i-4)}{6+11i-8}$

$= \frac{0+5i - (-1+i)}{-2+11i}$

$= \frac{6i-1}{-2+11i}$

25) $\frac{2-i}{1+i} - \frac{1-i}{1-i}$

$= \frac{(2-i)(1-i)}{(1+i)(1-i)} - 1$

$= \frac{2-2i-i+i^2}{1-i^2} - 1$

$= \frac{2-3i-1}{2} - 1$

$= \frac{1-3i}{2} - 1$

$= \frac{1-3i-2}{2} = \frac{-1-3i}{2}$

26) $\frac{1+2i}{2-i} - \frac{1-i}{1+i}$

$= \frac{(1+2i)(1+i) - (1-i)(2-i)}{(2-i)(1+i)}$

$= \frac{1+3i+2i^2 - (2-i+2i-i^2)}{2-i-i+i^2}$

$= \frac{1+3i-2 - (1-i+i)}{2-2i-1}$

$= \frac{-1+3i - (1-i)}{1-2i}$

$= \frac{-1+3i-1+i}{1-2i} = \frac{-2+4i}{1-2i}$

$= \frac{-2+4i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{(-2+4i)(1+2i)}{1-4i^2}$

$= \frac{-2-4i+4i+8i^2}{1+4} = \frac{-2-4i-8}{5} = \frac{-10-4i}{5}$

27) $\frac{1+2i}{1-i} - \frac{1-i}{1+i}$

$= \frac{(1+2i)(1+i) - (1-i)(1+i)}{(1-i)(1+i)}$

$= \frac{1+3i+2i^2 - (1-i^2)}{1-i^2}$

$= \frac{1+3i-2 - (1+1)}{2} = \frac{-1+3i-2}{2} = \frac{-3+3i}{2}$

28) $\frac{1+2i}{2-i} - \frac{1-i}{1+i}$

$= \frac{(1+2i)(1+i) - (1-i)(2-i)}{(2-i)(1+i)}$

$= \frac{1+3i+2i^2 - (2-i+2i-i^2)}{2-i-i+i^2}$

$= \frac{1+3i-2 - (1-i+i)}{2-2i-1}$

$= \frac{-1+3i - (1-i)}{1-2i}$

$= \frac{-1+3i-1+i}{1-2i} = \frac{-2+4i}{1-2i}$

$= \frac{-2+4i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{(-2+4i)(1+2i)}{1-4i^2}$

$= \frac{-2-4i+4i+8i^2}{1+4} = \frac{-2-4i-8}{5} = \frac{-10-4i}{5}$

29) $\frac{1+2i}{2-i} - \frac{1-i}{1+i}$

$= \frac{(1+2i)(1+i) - (1-i)(2-i)}{(2-i)(1+i)}$

$= \frac{1+3i+2i^2 - (2-i+2i-i^2)}{2-i-i+i^2}$

$= \frac{1+3i-2 - (1-i+i)}{2-2i-1}$

$= \frac{-1+3i - (1-i)}{1-2i}$

$= \frac{-1+3i-1+i}{1-2i} = \frac{-2+4i}{1-2i}$

$= \frac{-2+4i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{(-2+4i)(1+2i)}{1-4i^2}$

$= \frac{-2-4i+4i+8i^2}{1+4} = \frac{-2-4i-8}{5} = \frac{-10-4i}{5}$

30) $f(z) = (iz^2 + 3z^2)^3$

$f'(z) = 3(iz^2 + 3z^2)^2 \cdot \frac{d}{dz}(iz^2 + 3z^2)$

$= 3(iz^2 + 3z^2)^2 (2iz + 6z)$

$= 3(iz^2 + 3z^2)^2 (2z(i+3))$

$= 6z(iz^2 + 3z^2)^2 (i+3)$

$= 6z(i^2z^4 + 6iz^4 + 9z^4)(i+3)$

$= 6z(-z^4 + 6iz^4 + 9z^4)(i+3)$

$= 6z(8z^4 + 6iz^4)(i+3)$

$= 6z^5(8 + 6i)(i+3)$

$= 6z^5(8i + 24 + 6i^2 + 18i)$

$= 6z^5(8i + 24 - 6 + 18i)$

$= 6z^5(18i + 18)$

$= 108z^5(1+i)$

Name: Submit this sheet to Supervisor, before leaving the Examination Center.

Mth 632

Reduce $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$

in Real form

Express $-1+i$ in exponential form (3) marks

Express $\sqrt{3}-i$ in exponential form 2 marks

Mcqz

Mcqz are very very easy and mostly were 1-20 marks

Cauchy-Riemann rule

$$f'(z) = z - \bar{z}$$

5 marks

21) Solve $(-3+2i)^2 = (-3+2i)(-3+2i) = -3(-3+2i) + 2i(-3+2i)$
 $= 9 - 6(-3+2i) + (-6+4i) = 9 - 12i - 6 + 4i = 3 - 8i$

22) Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2+y^2}$

23) $-\sqrt{3} + i$ in exponential form $\frac{1}{1.73} = 0.577$
 $2 \cos \theta = -\sqrt{3}$ $\cos \theta = -1$
 $\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

24) $f(z) = (z+4i)^8$ at $z = 2+4i$
 $f'(z) = 8(z+4i)^7 \Rightarrow f'(2+4i) = 8(2+4i+4i)^7 = 8(2+8i)^7$
 $f''(z) = 56(z+4i)^6$
 $= 56(2+8i)^6 = 56(2^6 + 6 \cdot 2^5 \cdot 8i + \dots)$

25) $(-4+4i)^3$ Evaluate $(3+2i)^2$ at $3+2i$
 $(2+4i)^2 = 4 + 16i + 16i^2 = 4 + 32i - 16 = -12 + 32i$



$z = -\sqrt{3} + i$
 $z = re^{i\theta}$
 $r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$
 $\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
 $= 150^\circ$

$(3+2i)^2 = 9 + 12i + 4i^2 = 9 + 12i - 4 = 5 + 12i$
 $(2+4i)^2 = 4 + 16i + 16i^2 = 4 + 32i - 16 = -12 + 32i$

Math 632

Q12 Evaluate $(-4-4i)^3$

Q13 Express $(-3+i)$ in the form $a+ib$

Q14 $z = x+iy$

$$x = \frac{z + \bar{z}}{2} \quad y = \frac{z - \bar{z}}{2i}$$

Q15 $(1+z)^2 = 1 + \bar{z}^2 + 2z$ prove

① $\frac{3+i}{-1+2i}$
 $\frac{2+3i}{2+3i}$

② $\sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$

③ $z = 10^{i\theta}$
 $\frac{1}{z} = \frac{1}{10^{-i\theta}}$

④ $f(z) = \frac{3iz}{5}$
 $= \frac{3i \cdot 10^2}{5} = 6i$

⑤ $f(z) = \frac{3iz-i}{3}$
 $\left. \begin{aligned} 2z+i &= 0 \\ -i &= 2z \\ \frac{-i}{2} &= z \end{aligned} \right\} = \frac{3i(-i)-i}{3}$
 $= \frac{6i-i}{3} = 5i$

⑥ $\sqrt{6^2+7^2}$
 $= \sqrt{36+49}$
 $= \sqrt{85}$

⑦ $(4-i)^2$
 $= 16 + i^2 - 2 \cdot 4 \cdot i$
 $= 16 - 1 - 8i$
 $= 15 - 8i$

⑧ $f(z) = z^2 + z + 1$
 $= (i-1)^2 + i - 1 + 1$
 $= i^2 - 2i + 1 + i - 1 + 1$
 $= -2i + i + 1 = i(-2+1)$
 $= -i$

⑨ $f(z) = z^2 + 3z - 1$
 $= i^2 + 3i - 1$
 $= -1 + 3i - 1$
 $= -2 + 3i$

⑩ $f(z) = 2z^3 + 2z - 7$
 $f'(z) = 6z^2 + 2$

⑪ $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$

$= \frac{1+2i}{3-4i} \times \frac{3+4i}{3+4i} + \frac{2-i}{5i} \times \frac{i}{i}$
 $= \frac{(1+2i)(3+4i)}{(3-4i)(3+4i)} + \frac{2i-i^2}{5(-1)}$
 $= \frac{3+4i+6i+8i^2}{(3^2-(4i)^2)} + \frac{2i-(-1)}{5(-1)}$
 $= \frac{3+10i-8}{9+16} + \frac{2i+1}{-5}$
 $= \frac{-5+10i}{25} - \frac{2i+1}{5}$
 $= \frac{-5+10i-5(2i+1)}{25}$
 $= \frac{-5+10i-10i-5}{25}$
 $= \frac{-10}{25} = -\frac{2}{5}$

⑫ Limit $(x+y) \rightarrow (2+i) 3xy - y^2$
 $3 \cdot 3 \cdot 4 - 4^2$
 $= 36 - 16$
 $= 20$

⑬ Limit $z \rightarrow 2+i (z^2 - 4z + 2 + 5i)$
 $= (2+i)^2 - 4(2+i) + 2 + 5i$
 $= 4 + i^2 + 4i - 8 - 4i + 2 + 5i$
 $= 4 - 1 + 4i - 8 - 4i + 2 + 5i$
 $= -3 + 5i$

$= \frac{-5+10i-8}{9+16} + \frac{2i+1}{-5}$
 $= \frac{-13+10i}{25} - \frac{2i+1}{5}$
 $= \frac{-13+10i-5(2i+1)}{25}$
 $= \frac{-13+10i-10i-5}{25}$
 $= \frac{-18}{25}$

The number $z = x + iy$ can be written in polar form as ANS: $r \cos \theta + i r \sin \theta$

Q3. If $z = e^{\theta}$, then $z' =$ ANS: e^{θ}

Q4. The absolute value of complex number $Z = 1 - i$ is $\sqrt{2}$

The absolute value of complex number $Z = a + ib$ is 1) $a^2 + b^2$, 2) $a + b$, 3) a^2 , 4) $\sqrt{a^2 + b^2}$

The function $f(z) = \frac{1}{z^2 + 1}$ is defined everywhere in the finite plane except at the point 1) $z = \pm 1$, 2) $z = \pm i$, 3) $z = \pm i$,

4) $z = \pm 2i$

Let a and b denote complex constants, then $\lim_{z \rightarrow \infty} az + b =$ 1) $az + b$, 2) $az + b$, 3) $a + b$, 4) b

The magnitude of $\exp(2 + 3i)$ is 1) $\exp(2)$, 2) $\exp(3)$, 3) $\exp(2 + 3)$, 4) $\exp(2/3)$

square of $1 + 2i$ is $-3 + 4i$

Q10. Let $f(z) = \frac{1}{z}$ is discontinuous at $z =$ 1) 0 , 2) 1 , 3) -1 , 4) i

Q11. Let $f(z) = \frac{1}{z^2 + 1}$ is discontinuous at $z =$ 1) $\pm i$, 2) ± 1 , 3) ± 2 , 4) $\pm 2i$

Q12. Let $f(z) = \frac{1}{z - 3i}$ is discontinuous at $z =$ 1) $3i$, 2) $-3i$, 3) i , 4) $-i$

The polar Cauchy - Riemann equation on a pair of real - valued function of two real variables u and v are 1) $u_x = v_y$ and $u_y = -v_x$, 2) $u_x = v_y$ and $u_y = v_x$, 3) $u_x = v_y$ and $u_y = -v_x$, 4) $u_x = v_y$ and $u_y = v_x$

The limit of $f(z) = z^2 - 2z + 1$ at $z = i + 1$ is 1) 1 , 2) -1 , 3) i , 4) $-i$

Q15. The limit of $f(z) = z^2 + 3z - 1$ at $z = i$ is 1) $3 + 2$, 2) $3i$, 3) $2i - 3$

Q16. Let $f(z) = 12z^2 + 15z - 30$ then $f'(z)$ is 1) $z + 15$, 2) $12z^2 + 15$, 3) $36z^2 + 15$, 4) $36z^2 + 15$

Multiply $w = (3 + 2i)$ with $z = (1 + 7i)$ we get $z.w =$ 1) $3 + 14i$, 2) $11 + 23i$, 3) $21 + 14i$, 4) $21 + 2i$

conjugate of complex numbers $z = -3 - 3i$ is 1) $-3 + 3i$, 2) $3 + 3i$, 3) $-3 + 6i$, 4) $3 - 3i$

The sum of two complex numbers $z = 3 + i$, $w = -1 + 2i$ is $z + w =$ 1) $2 + 3$, 2) $3 + 2$, 3) $-1 + i$

Q20. Solve $(-3 + 2i)^2$ (2)

Q21. Compute the exponential form of $\sqrt{3} - i$ (2)

Q22. Verify that $(3, 1)(3, -1) \left(\frac{1}{5}, \frac{1}{10} \right) = (2, 1)$ (3)

Q23. By using Cauchy - Riemann equations (in polar form), find $f'(z)$ of the $f(z) = \frac{1}{z^2} = \frac{1}{r^2} (\cos 4\theta - i \sin 4\theta)$ (3)

Q24. Using the definition $\frac{dw}{dz} = \lim_{\Delta z \rightarrow 0} \frac{\Delta w}{\Delta z}$, prove that $\frac{dw}{dz} = -\frac{1}{z^2}$ when $w = \frac{1}{z}$ ($z \neq 0$). (5)

Evaluate, $\lim_{z \rightarrow i} \frac{z^2 + z - 2 + i}{z^2 - 2z + 1}$ (5)

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